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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report summarizes the research activity in the following research areas: eigenfunctions expansions associated with elliptic operators; best data approximation; and multivariate splines. Results were published in 21 papers in research journals and conference proceedings. Fourteen of the papers are on elliptic and abstract operators, two on best data approximation, and five on splines.

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STABLE SUMMABILITY AND APPROXIMATION THEORY

FINAL REPORT

LOUISE A RAPHAEL

JULY 25, 1989

U.S. ARMY RESEARCH OFFICE

GRANT : DAAG-29-84-G0004

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HOWARD UNIVERSITY

WASHINGTON, DC 20059

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Final Report

Introduction: This report summarizes the research activity of the principal investigator Louise A. Raphael of Howard University, during the period of July 1984 through June 1989 on Army Research Office - Division of Applied Mathematics grant DAAG-29-84-G0004/P-21235-MA-H, in the following research areas:

* eigenfunction expansions associated with elliptic operators;
* best data approximation; and
* multivariate splines.

A two year no-cost extension was granted by ARO, so that Raphael could accept a rotator program directorship at the National Science Foundation during the 1986-88 academic years. While no ARO funds were spent during this period, Raphael continued to publish in mathematical areas of interest to ARO.

Raphael gratefully acknowledges that the research and administrative experiences gained under the aegis of ARO has enabled her to participate in cutting edge mathematical research and to have been part of the mathematical infrastructure at the NSF for two years. She hopes that other faculty members at HBCU and institutions with large min[illegible]y student populations can also benefit from ARO programs.

Research:

During the five year grant period, Raphael completed more research than proposed. Her results are published in 21 papers in research journals and conference proceedings. Fourteen of the papers are on elliptic and abstract operators, two on best data approximation, and five on splines. Of her 16 collaborative publications, she worked with the following mathematicians:

* Robert Carroll, University of Illinois
* Charles Chui, Texas A & M University and ARO researcher
* Harvey Diamond, West Virginia University
* Mark Kon, Boston University/Columbia University
* Alexander Ramm, Kansas State University
* James Young, MIT.

Elliptic Operators

The 14 differential and abstract operator papers determine pointwise and L^p-convergence relationships between eigenfunction expansions for certain classes of elliptic operators with continuous spectrum on $\mathbb{R}^n$, $n \geq 1$; and between abstract operators of which elliptic operators are an illustration.

A typical example of her results for two uniformly elliptic operators sharing the same leading terms [$\sum_{|s| \leq m} a_s D^s$ *is* *uniformly elliptic* *if its* leading symbol $a(x,z) = \sum_{|s| \equiv m} a_s(x) z^s$ satisfies $c_1|z|^m \leq a(x,z) \leq c_2|z|^m$ ($z \in \mathbb{R}^n$) with constants $c_1, c_2 > 0$ independent of $x \in \mathbb{R}^n$.] states that their eigenfunction expansion behave in essentially the same way under analytic summability techniques. This means that this class of eigenfunction expansions, including the Schrodinger operators, behave like the Fourier transform on $\mathbb{R}^n$.

Analogues of these equisummability results were applied to:

* Sturm-Liouville system [2,8]
* heat equation [2,11]
* Schrodinger and Dirac operators [8,10]
* Fourier integrals [3,11], and
* abstract operators [4,6,14].

Moreover in papers [1,5,7,9,10] these analytic weighted eigenfunction expansions recovered functions from noisy data. In paper [12] these techniques were applied to the one-dimensional seismic wave problem.

The published papers are in the ARO library. Abstracts were provided in the biannual ARO reports. These are:

1. With H. Diamond, M. Kon. "A Regularization of the Pointwise Summation of Singular Sturm-Liouville Eigenfunction Expansions". Journal of Approximation Theory and Its Applications. Vol. 1, No. 5, 1985, pp. 61-69.

2. "Pointwise Equisummability of Elliptic Operators". Proceedings of the Workshop on Spectral Theory of Sturm-Liouville Differential Operators, Argonne National Laboratory. ANL-84, 73, Dec. 1984, pp. 181-185.

3. "Resolvent Means and Inverting Generalized Fourier Transforms". Canadian Journal of Mathematics, Vol. XXX VIII, No. 4, 1986, pp. 861-877.

4. "Equisummability of Eigenfunction Expansions under Analytic Multipliers". Journal of Mathematical Analysis and Applications, Vol. 115, No. 1, pp. 93-104, 1986.

5. With H. Diamond, M. Kon. "On the Pointwise Regularization of Elliptic Eigenfunction Expansions". Bulletin of London Mathematical Society. Vol. 18, 1986, pp. 44-50.

6. With M. Kon, A. Ramm. "Equisummability for Linear Operators in Banach Spaces". Proceedings of the Royal Edinburgh Society, Vol. 106A, 1987, pp. 315-325,

7. With H. Diamond, M. Kon. "Stable Summations Methods for Elliptic Eigenfunction Expansions", Transactions of Third Army Conference on Applied Mathematics and Computing, 86-1, 1988 pp. 819-824.

8. With M. Kon, J. Young. "Kernels and Equisummation Properties of Uniformly Elliptic Operators". Journal of Differential Equations, 67, 1987, pp. 256-268.

9. "Pointwise and $L^p(\mathbb{R}^n)$ Convergence of Elliptic Eigenfunction Expansions". Proceedings of International Workshop on Applied Differential Equations, Beijing, China. Editors: Xiao Shutie and Pu Fuquan, World Scientific Publ., 1986, pp. 138-151.

10. With H. Diamond, M. Kon. "On Recovering Functions from Noisy Data". Approximation Theory V, Editors: C. Chui, L. Schumaker and J. Ward, Academic Press, 1986, pp. 323-326.

11. With M. Kon, J. Young. "On Relating Generalized Expansions to Fourier Integrals". Differential Equations and Mathematical Physics, Editors I. Knowles and Y. Saito, Springer Verlag, 1987, pp. 248-255.

12. With R. Carroll, "Some Inverse Problems with Spectral Limitations on Data". SIAM Journal of Applied Mathematical Analysis, Vol. 48, No. 5, 1988, pp. 1194-1205.

13. With M. Kon. "Some Negative L^p results for Eigenfunction Expansions Associated with Scattering Theory". Oscillation, Bifurcation and Chaos, Proceedings of the 1986 Annual Seminar, AMS,1987, pp. 221-226

14. With M. Kon, A. Ramm. "Applications of Operator Approximation Theorems to Fourier Analysis". Constructive Theory of Functions, 1988, Bulgarian Academy of Sciences, pp. 276-282.

Data Approximation:

The two papers [15,16] on best data approximation prove that if there is a unique element g of a subspace V of a normed linear space $(X,\| \|) = \{f \mid f: \mathbb{R}^n \to \mathbb{R}\}$, which matches the derivatives at zero of any given function in C^{m+1} up through all derivatives of order m, then the best local approximant of an f in C^{m+1}, with respect to the L^p norm, at $\vec{x} = \vec{0}$ exists and is equal to the function g in V. These results were applied to rational and quasi-rational approximants. The results appear in the following papers (which are in the ARO library):

15. With C. Chui, H. Diamond. "Best Local Approximation in Several Variables". Journal of Approximation Theory, Vol. 40, No. 4, 1984, pp. 343-350.

16. With C. Chui, H. Diamond. "On Best Data Approximation". Journal of Approximation Theory and Its Applications, Vol. 1, No. 1, 1984, pp. 37-56.

Splines:

Four of the next five papers are in the new (last ten years) and active area of multivariate splines. Two [18,19] deal with interpolation schemes guaranteeing local optimal order of approximation to sufficiently smooth the

data function on regular and non-regular rectangular grids. Two [20,21] are devoted to the study of shape-preserving approximation, quasi-interpolation and interpolation of functions by box spline surfaces. Shape-preserving means preserving convexity, monotonicity, and positivity. The spline papers (also in the ARO library) are

17. "A Jackson-Type Theorem by Averaging Splines Bounding a Class of Differential Functions". Journal of Approximation Theory, Vol. 43, No. 2, 1985, pp. 124-313.

18. With C. Chui, H. Diamond. "Order of Approximations of Interpolating Bivariate Quadratic B-splines". Transactions of the Fourth Army Conference on Applied Mathematics and Computing. ARO 87-1, 1987 pp. 1261-1266.

19. With C. Chui, H. Diamond, "Interpolation by Multivariate Splines. Mathematics of Computations, Vol. 51, No. 183, 1988, pp. 203-218.

20. With C. Chui, H. Diamond. "Convexity Preserving Quasi-interpolation and Interpolation by Box Spline Surfaces", Transactions of the Fifth Army Conference on Applied Mathematics and Computing, 88-1, 1988, pp. 301-310.

21. With C. Chui, H. Diamond. "Shape Preserving Quasi-interpolation and Interpolation by Box-Spline Surfaces". Journal of Computational and Applied Mathematics, vol. 25, 1989, pp. 169-198.

New Research: Splines and Partial Differential Equations.

Many researchers recognize the advantages of using splines to construct surfaces to fit given data points. However, little has been done to exploit the calculus of the box splines to find numerical solutions to non-linear partial differential equations.

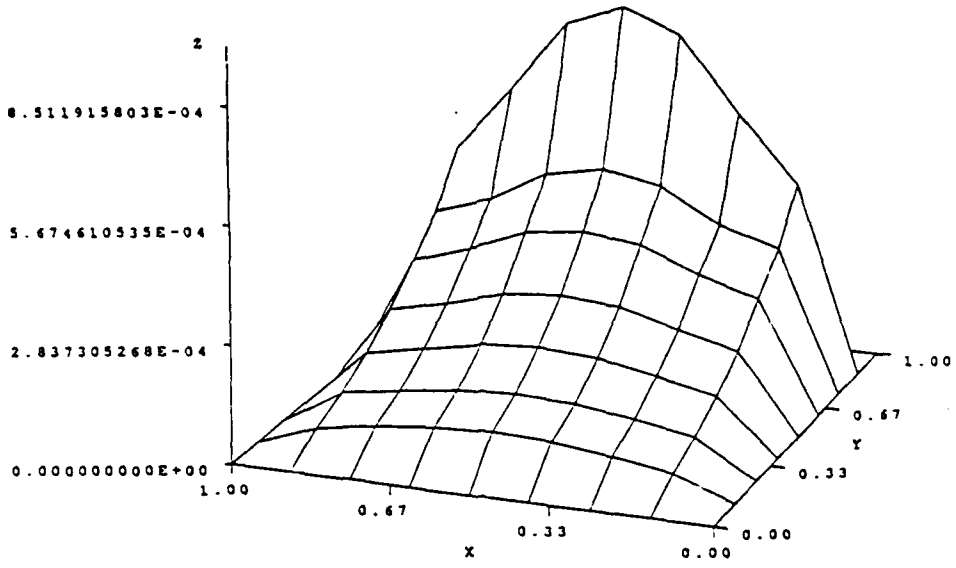
Harvey Diamond of West Virginia University, Daniel Williams of Howard University and Raphael have formulated a Galerkin method for Laplace's equation using C^1 bivariate quadratic splines on an uniform grid. We compared our results with a five point finite difference discretization with approximation order $O(h^2)$ [see Appendix pages 8-9].

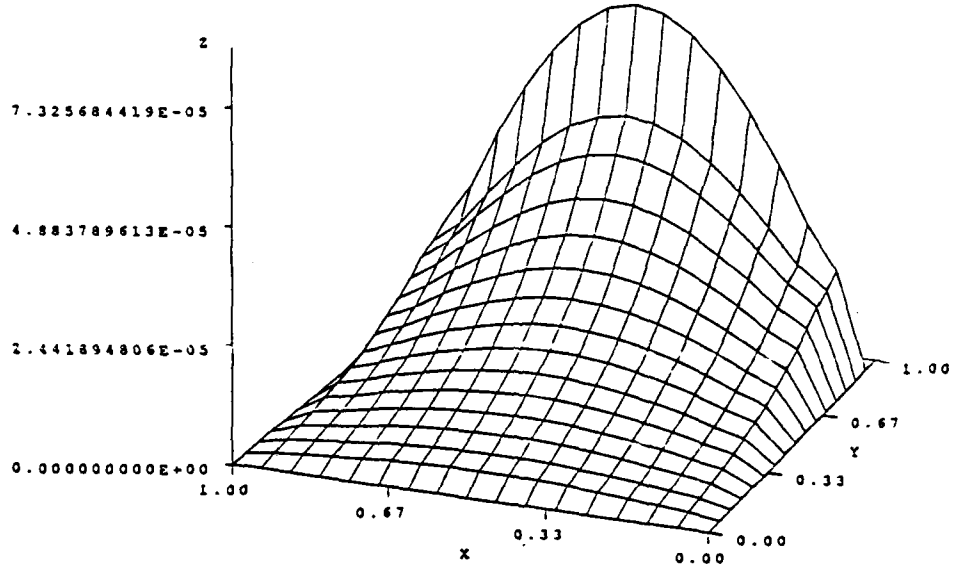
Based on this prototypical model we plan to develop a class of computationally, high order adaptive numerical methods for partial differential equations. By interlacing the box spline calculus with Galerkin and multigrid methods, we will construct computer codes which allow the rapid formulation and solution of numerical schemes for PDE's for any desired box spline space and order of approximation. We will test the performance of our schemes against reliable numerical data for the driven cavity flow problem in nonlinear fluid dynamics such as the seminal Schreiber-Keller paper.

Projection: Under ARO support, Raphael has enjoyed her most productive and creative research period. Her current research in applying box splines to non-linear PDE's may have applications to:

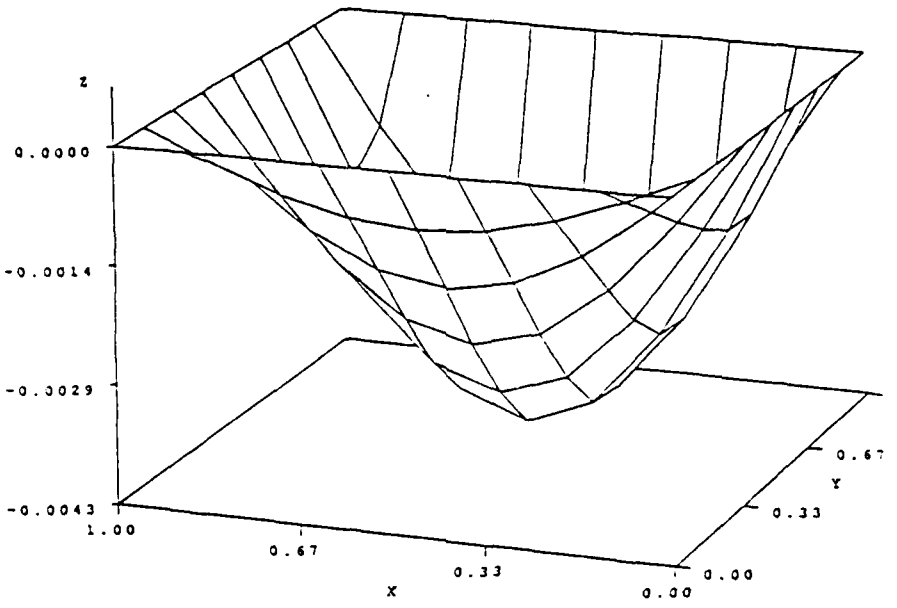
* time evolution of magnetization of the Landau-Lefschitz equations;
* construction of airplane surfaces to determine aerodynamic performance properties; and
* robot path generation.

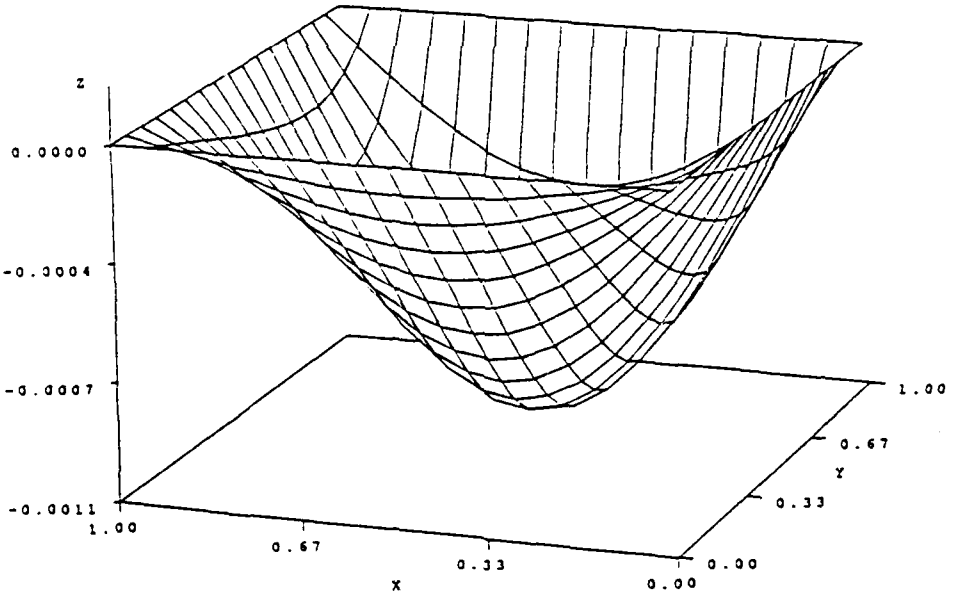
Raphael is truly excited about her research in this area and is grateful to ARO's Division of Applied Mathematics for the encouragement, collegiality, assistance and support they have consistently given her during the years of her grants.





Error in computed solution of Laplace's equation using box spline formulation.
Top figure: h=1/8 ; Bottom figure: h=1/16.
Maximum error decreases by a factor of about 1/10.





Error in computed solution of Laplace's equation using 5-point finite difference discretization with approximation order $O(h^2)$.
Top figure: h=1/8; Bottom figure: h=1/16.
Maximum error decreases by factor of about 1/4.